Unbalanced Growth in a Small Economy with Open Capital Markets

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Abstract This paper shows that in an economy with open capital markets, there exists a force toward unbalanced growth driven by intertemporal substitution of consumption and trade imbalance. We construct a two-sector Ramsey economy composed of a technologically progressive tradable sector and a technologically stagnant nontradable sector. We show that a capital account liberalization in the economy will drive the long-run share of the tradable sector toward one or zero, depending on whether the autarky interest rate is higher or lower than the world interest rate. The difference between the autarky interest rate and the world interest rate ultimately dictates the direction of structural transformation regardless of the progression of the Baumol's disease that pushes up the relative price and the share of nontradables. Because the autarky interest rate is increasing in the rate of technological progress, our result suggests that cross-country differences in the rate of technological progress may be an important factor in accounting for the diverse patterns of structural change in a financially integrated world.

Keywords capital account liberalization; structural transformation; Baumol's disease; Balassa-Samuelson effect; trade imbalance

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1. INTRODUCTION

Structural transformation—the unbalanced growth of a few major sectors of the economy—has long been an important part of research on growth theory. Most advanced economies in the world experienced falling shares of agriculture and rising shares of manufacturing and services, both in terms of value-added and employment, at an earlier phase of development. Thereafter, they experienced falling shares of manufacturing and rising shares of services. The phenomenon had early been documented by economic historians and development economists (e.g., Clark, 1940; Chenery, 1960; Kuznets, 1966). Recently, a number of papers attempted to explain it formally based on the neoclassical growth theory. These studies largely follows two lines. One line of research, influenced by Engel (1985), explains unequal sectoral growth rates based on different income elasticities across consumption goods (e.g., Echevarria, 1997, Kongsamut, Rebelo, and Xie, 2001, and Foellmi and Zweimuller, 2008). Another line of research centers on the prediction of Baumol (1967) that the share of services would rise toward one when productivity growth is slower in services than in manufacturing, and manufactures and services are poor substitutes in consumption. Ngai and Pissarides (2007) and Acemoglu and Guerrieri (2008) are influential examples for this line of research.

Most works on structural transformation assume autarkic economies. There are relatively few works explaining structural transformation from an international viewpoint. This is surprising given that many concerns about deindustrialization are voiced in the context of international competition. Further, the loss of manufacturing jobs is often attributed to not just trade, but trade deficits with rival countries, as is exemplified by the protectionism of US President Donald Trump against China. The purpose of this paper is to show that in an economy with open capital markets, there exists a force toward unbalanced growth operating through the channel of trade imbalances. In a neoclassical small open economy, capital account liberalization drives a wedge between consumption growth rate and productivity growth rate. If the autarky interest rate is higher than the world rate, a capital account liberalization lowers the interest rate and

flattens consumption path relative to output path, putting upward pressure on consumption-output ratio in the short run and downward pressure in the long run. The shift of consumption profile induces the economy to borrow from the world in earlier periods and service resulting debts by running trade surpluses in later periods. In an economy composed of a tradable sector and a nontradable sector, the only way to generate trade surpluses is to shift resources from the nontradable sector to the tradable sector, raising the share of the latter in the long run. When the autarky interest rate is lower than the world interest rate, the opposite results would prevail. Because the autarky interest rate is increasing in the rate of technological progress, this result suggests that an economy whose rate of technological progress is high relative to others would experience a long-run increase in the share of the tradable sector after a capital account liberalization. In contrast, an economy with relatively slow progress of technology would experience a long-run decrease in the tradables share.

We derive the result using a small open economy version of a two-sector Ramsey model. We examine an economy that produces tradables and nontradables that are poor substitutes in consumption, and where the total factor productivity grows faster in the tradable sector than in the nontradable sector. In this economy, the relative price of nontradables increases without bound, and the share of nontradables in consumption expenditure rises toward unity: "the Baumol's cost disease" emerges. We show that a lower interest caused by a capital account liberalization will drive the economy toward complete industrialization in the long run, despite the presence of the Baumol's disease that pushes the economy toward deindustrialization on the consumption side.

This paper shares the motivation of trade theorists who showed that the predictions derived from a closed economy model can be overturned in an interdependent world. In an economy with open goods markets, a sector whose technology improves faster than other sectors can expand rather than shrink, contrary to Baumol's prediction. The sector can acquire comparative advantage and increase its exports, drawing labor and capital from the other sectors. Findlay and Grubert (1959) long before showed the possibility in a simple two-good model. Matsuyama (2009) demonstrated the mechanism clearly in a three-sector static

economy with non-homothetic preferences and nontradables production. Uy, Yi, and Zhang (2013) employed a multi-sector dynamic simulation model to show that trade can explain the sustained expansion of manufacturing share in South Korea. While our model also emphasizes interdependence as a source of unbalanced growth, the underlying mechanism is totally different from the trade-based models. In our model, structural change is driven by international capital mobility and intertemporal substitution of consumption through overall trade imbalances, not by static exchanges of goods based on comparative advantages.

In this respect, we believe that our study adds a novel international dimension to the growing literature on structural transformation. However, our model is missing some important elements necessary to account for reality. First of all, the responses of our economy to a capital account liberalization are too drastic. The economy either goes toward complete industrialization or complete deindustrialization, depending on whether its productivity is growing faster or slower than other countries. The extreme responses could be muted in more realistic models incorporating borrowing constraints, finite horizon, or technology convergence across countries. In addition, our model cannot resolve "the allocation puzzle" identified by Gourinchas and Jeanne (2013). As in the standard neoclassical economy, in our model too, capital tends to flow into countries with faster technological progress, contrary to their finding. We need to introduce some "wedges" in saving and investment to catch distortions present in international capital flows. ¹ Because of these limitations, we do not attempt to provide rigorous empirical support for our theory. However, we note that our theory may be related to the phenomenon of "premature deindustrialization". Rodrik (2016) finds that compared to advanced countries that industrialized earlier, recent industrializing countries tended to arrive at a peak manufacturing share at a lower income level, and the level of the peak tended to be lower than before. He also presents evidence that fast growing Asian countries largely evaded these trends, while Latin American countries were hardly hit. He attributes it to trade global-

¹However, Reinhardt, Ricci and Tressel (2013) find that capital do flow into lower income countries and out of higher income countries among financially open economies. Their findings could be consistent with our model if the initial income levels of countries are negatively correlated with their future productivity growth rates in the sample of financially open countries.

ization after 1990s. Our paper suggests that financial globalization may also be responsible for the premature deindustrialization of slow growing countries.

We do not present any novel model for our theoretical investigation. Our economy can be thought of as a two-sector version of the multi-sector structural change model of Ngai and Pissarides (2007) ². We open up their economy and contrast their closed economy results with our open economy ones. Our model is closely linked to the literature on the Balassa-Samuelson effects. Indeed, our model is identical to the dependent economy model presented by Obstfeld and Rogoff (1996) in their textbook. The difference is that we completely solve the model for balanced growth path in the case where a country's long-run rate of technological progress is different from that of the world. This paper also complements Kim, Oh, and Song (2019), which derives similar results under very restrictive assumptions. Here, we analyze balanced growth path equilibria in a more general setting that allows more general preferences, more general production structure, and most importantly, the Baumol's disease.

This paper is organized as follows. Section 2 sets up the model. Section 3 derives the long-run equilibrium for an autarkic economy. Section 4 analyzes the dynamics for a small open economy. Section 5 briefly concludes the study.

2. THE MODEL

The economy has a representative household that solves the following problem.

$$\max \int_0^\infty \frac{\tilde{c}^{1-\rho} - 1}{1-\rho} \exp\left[-(\theta - n)t\right] dt$$
s.t. $\tilde{a} = (r-n)\tilde{a} + W - P_T \tilde{c}_T - P_N \tilde{c}_N,$ (1)

$$\lim_{t\to\infty}\tilde{a}\exp\left[-\int_0^t(r(s)-n)ds\right]\geq 0,$$

²Our model is slightly more general than theirs in that we allow different capital intensities across sectors and non-logarithmic utility functions. In addition, we present a sharper characterization of balanced growth path for the two-sector case.

where

$$\tilde{c} = \left[(1 - \gamma)^{\frac{1}{\varepsilon}} \tilde{c}_T^{\frac{\varepsilon - 1}{\varepsilon}} + \gamma^{\frac{1}{\varepsilon}} \tilde{c}_N^{\frac{\varepsilon - 1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon - 1}}$$
(2)

 \tilde{c} denotes an index for consumption per capita, $1/\rho$ the intertemporal rate of substitution, θ the subjective discount rate, and n a constant rate of population growth. \tilde{a} represents asset holdings per capita. We use r for the interest rate, and W for the wage rate. $P_i(i = T \text{ or } N)$ denotes the price of good i. T refers to tradables, and N nontradables. Consumption index \tilde{c} is determined by the CES function in (2), where \tilde{c}_i is consumption per capita of good i. From now on, we will assume that $P_T = 1$. Thus all values are measured in units of tradables. The time dependence of variables is omitted for simplicity.

The supply side of the economy is characterized by the following equations.

$$Q_T = (A_T L_T)^{1 - \alpha_T} K_T^{\alpha_T}, \tag{3}$$

$$Q_N = (A_N L_N)^{1 - \alpha_N} K_N^{\alpha_N}, \tag{4}$$

$$\frac{\dot{A_T}}{A_T} = g_T,\tag{5}$$

$$\frac{\dot{A_T}}{A_T} = g_T,$$

$$\frac{\dot{A_N}}{A_N} = g_N.$$
(5)

 Q_i denotes the aggregate output of good i, and L_i and K_i are labor and capital employed in sector i. We assume that both goods are produced by Cobb-Douglas technologies. We use g_i to denote a constant rate of labor-augmenting technological progress in sector i.

We impose the following restrictions on parameter values.

Assumption 1.

$$g_T > g_N$$
.

Assumption 2.

$$\varepsilon$$
 < 1.

Assumption 3.

$$\alpha_T > \alpha_N$$
.

Assumption 4.

$$\theta - n > (1 - \rho) [\alpha_N g_N + (1 - \alpha_N) g_T].$$

Assumptions 1 and 2 are the drivers of Baumolian dynamics and are essential for generating the Baumol's cost disease: the situation where the relative price and the expenditure share of technologically stagnant sectors secularly increase³. As we will see shortly, when technology improves faster in the tradable sector than in the nontradable sector, the relative price of nontradables forever increases. With the ever-rising relative price of nontradables, their expenditure share in consumption increases perennially when tradables and nontradables are poor substitutes in consumption, or $\varepsilon < 1$.

Assumption 3 is not critical for our results below, but we adopt it to avoid taxonomy and because it is more plausible than the other case. Assumption 4 is a technical assumption necessary to prevent the objective function of the household from going unbounded.

The economy should satisfy the following resources constraints.

$$K_T + K_N = K, (7)$$

$$L_T + L_N = L, (8)$$

$$\frac{\dot{L}}{L} = n. \tag{9}$$

K and *L* are aggregate capital and labor, respectively. Capital depreciations will be ignored for simplicity.

Let $\tilde{e} = \tilde{c}_T + P_N \tilde{c}_N$ be the consumption expenditure per capita. Then, the ideal price index for \tilde{c} is given by:

$$P = \left[(1 - \gamma) + \gamma P_N^{1 - \varepsilon} \right]^{\frac{1}{1 - \varepsilon}}, \tag{10}$$

and

$$\tilde{e} = P \, \tilde{c},\tag{11}$$

$$s_N \equiv \frac{P_N \tilde{c}_N}{\tilde{c}} = \gamma \left(\frac{P_N}{P}\right)^{1-\varepsilon}.$$
 (12)

³De Gregorio, Giovannini, and Wolf (1994), Nordhaus (2008), and Hartwig (2011) find supporting evidence in the OECD and U.S. data.

 s_N is the expenditure share of nontradables in consumption, and is increasing in P_N . We define that $\Pi_N = \frac{\dot{P}_N}{P_N}$, and $\Pi = \frac{\dot{P}}{P}$. Then we can show that

$$\Pi = s_N \Pi_N, \tag{13}$$

$$\dot{s}_N = (1 - \varepsilon) s_N (1 - s_N) \Pi_N. \tag{14}$$

We can also derive the following necessary and sufficient conditions for the optimal consumption path.

$$\frac{\dot{\tilde{c}}_T}{\tilde{c}_T} = \frac{1}{\rho} \left(r - \theta - s_N \Pi_N \right) + \varepsilon s_N \Pi_N, \tag{15}$$

$$\frac{\dot{c}_N}{\tilde{c}_N} = \frac{1}{\rho} \left(r - \theta - s_N \Pi_N \right) - \varepsilon \left(1 - s_N \right) \Pi_N, \tag{16}$$

$$\frac{\dot{\bar{c}}}{\bar{c}} = \frac{1}{\rho} \left(r - \theta - s_N \,\Pi_N \right),\tag{17}$$

$$\lim_{t\to\infty}\tilde{a}\exp\left[-\int_0^t (r(s)-n)\,ds\right]=0. \tag{18}$$

By (13), $s_N\Pi_N$ is the inflation rate of the consumer price index, and thus r $s_N\Pi_N$ is the real interest rate in terms of CPI. Thus, (17) states the standard Ramsey condition for optimal consumption. Note that the growth rate of tradable consumption is higher than that of nontradable consumption by $\varepsilon\Pi_N$. This term captures substitution between two consumption goods in response to an increase in the relative price of nontradables. (18) is the transversality condition.

We assume that all markets in the economy are competitive. Suppose both tradables and nontradables are produced in the economy. Cost minimization implies that

$$1 = \phi_T A_T^{-(1-\alpha_T)} W^{1-\alpha_T} r^{\alpha_T}, \tag{19}$$

$$P_N = \phi_N A_N^{-(1-\alpha_N)} W^{1-\alpha_N} r^{\alpha_N}, \qquad (20)$$

$$\frac{K_T}{L_T} = \frac{\alpha_T}{1 - \alpha_T} \frac{W}{r},$$

$$\frac{K_N}{L_N} = \frac{\alpha_N}{1 - \alpha_N} \frac{W}{r}.$$
(21)

$$\frac{K_N}{L_N} = \frac{\alpha_N}{1 - \alpha_N} \frac{W}{r}.$$
 (22)

(19) states that the price of tradables must be equal to the unit cost of tradables in competitive equilibrium. (20) is the corresponding condition for nontradables. ϕ_i is a constant equal to $\alpha_i^{-\alpha_i}(1-\alpha_i)^{-(1-\alpha_i)}$. (21) and (22) are the conditions for cost minimization in the two sectors. Let $\lambda = L_N/L$, $n_T = \dot{L}_T/L_T$, $n_N = \dot{L}_N/L_N$. Then,

$$\frac{K}{L} = (1 - \lambda) \frac{K_T}{L_T} + \lambda \frac{K_N}{L_N},\tag{23}$$

$$n = (1 - \lambda) n_T + \lambda n_N. \tag{24}$$

The capital-labor ratio of the economy must be equal to a weighted average of capital intensities in the two sectors. Population growth rate must be equal to a weighted average of employment growth rates in the two sectors. In both equations, weights are given by the employment shares of the two sectors.

3. BALANCED GROWTH IN AUTARKY

Using the terminology of Herrendorf et al. (2013), we will anchor our analysis around approximate generalized balanced growth path (hereafter, AGBGP). We define the economy to be on a generalized balanced growth path (hereafter, GBGP) if r is constant, and L_i , K_i , Q_i , and $\tilde{c}_i L$ grow at constant, but possibly different, rates.⁴ In the following analysis, our economy will go infinitely close to a GBGP but it can never be exactly on the path. In this case, we define the economy to be on an AGBGP. The interest rate is asymptotically constant, but can never be exactly constant. The growth rate of each major variable converges to a constant value, but it can never be exactly constant. The enlarged definition of balanced growth path is useful for capturing unbalanced growth.

The following normalization of variables is used in the analysis below.

$$e \equiv \tilde{e}/A_T, \quad a \equiv \tilde{a}/A_T, \quad c_T \equiv \tilde{c}_T/A_T, \quad c_N \equiv \tilde{c}_N/A_T, \quad c \equiv \tilde{c}/A_T,$$
 $k_T \equiv K_T/(L_T A_T), \quad k_N \equiv K_N/(L_N A_T), \quad k \equiv K/(L A_T), \quad w \equiv W/A_T,$ $p_N \equiv P_N/\left(\frac{A_T}{A_N}\right)^{1-\alpha_N}.$

⁴On a balanced growth path, major quantity variables grow at an identical rate.

In autarky, both goods should be produced, and (19) and (20) must hold. Rewriting (19) through (23) using normalized variables,

$$1 = \phi_T w^{1 - \alpha_T} r^{\alpha_T}, \tag{25}$$

$$p_N = \phi_N w^{1 - \alpha_N} r^{\alpha_N}, \tag{26}$$

$$k_T = \frac{\alpha_T}{1 - \alpha_T} \frac{w}{r},\tag{27}$$

$$k_N = \frac{\alpha_N}{1 - \alpha_N} \frac{w}{r},\tag{28}$$

$$k = (1 - \lambda) k_T + \lambda k_N. \tag{29}$$

In addition, autarky requires that domestic demand must be equal to domestic supply in each sector.

$$\tilde{c}_T L + \dot{K} = Q_T = (A_T L_T)^{1 - \alpha_T} K_T^{\alpha_T},$$
(30)

$$\tilde{c}_N L = Q_N = (A_N L_N)^{1 - \alpha_N} K_N^{\alpha_N}. \tag{31}$$

Using normalized variables and the definition of λ ,

$$c_T + \dot{k} + (n + g_T)k = (1 - \lambda)k_T^{\alpha_T},$$
 (32)

$$c_N = \left(\frac{A_T}{A_N}\right)^{-(1-\alpha_N)} \lambda k_N^{\alpha_N}. \tag{33}$$

(30) and (31) contain an important restriction: tradables are used for both consumption and investment, but nontradables are used only for consumption. We will later discuss in detail the important implications of this restriction. In autarky, assets are entirely composed of domestic assets, and \tilde{a} must be equal to K/L. Thus, the transversality condition (18) can be rewritten as:

$$\lim_{t\to\infty} k \exp\left[-\int_0^t \left(r(s) - n - g_T\right) ds\right] = 0.$$
 (34)

Using (25) through (28), we can express w, p_N , k_T , and k_N as functions of r alone. Explicit expressions for them can be found in Appendix A. On an AGBGB, r converges to a constant. Thus, all these variables should also converge to constant values. In the following, we will denote the asymptotic value of v by v^* , and the growth rate of v by g_v .

The constancy of p_N implies that P_N grows at the rate of $(1 - \alpha_N)(g_T - g_N)$. The relative price of nontradables increases to infinity on an AGBGP, and by (12), the expenditure share of nontradables in consumption approaches unity when $\varepsilon < 1$. Combining these results with (16) and (33), we obtain

$$g_{c_N}^* = \frac{1}{\rho} \left(r^* - \theta - (1 - \alpha_N) \left(g_T - g_N \right) \right) - g_T = n_N^* - n - (1 - \alpha_N) \left(g_T - g_N \right)$$
 (35)

The constancy of r^* implies the constancy of $g_{c_N}^*$ and n_N^* . It follows from (15) and (35) that

$$g_{c_T}^* = \frac{1}{\rho} (r^* - \theta) + \left(\varepsilon - \frac{1}{\rho} \right) (1 - \alpha_N) (g_T - g_N) - g_T$$

= $n_N^* - n - (1 - \varepsilon) (1 - \alpha_N) (g_T - g_N)$. (36)

Because $\varepsilon < 1$ and n_N^* cannot be greater than n—employment in one sector cannot forever grow faster than population, $g_{c_T}^*$ and $g_{c_N}^*$ are strictly negative. Thus, c_T and c_N must approach zero on an AGBGP.

We now prove that (32) and (34) imply that on an AGBGP, employment in the two sectors should asymptotically grow at the rate of n. To prove this, suppose that $n_N^* < n$. Then $\lambda^* = 0$. (29) implies that k converges to k_T^* , which is a constant. Thus, k goes to zero. It follows from (32) that $n + g_T = (k_T^*)^{\alpha_T - 1}$ or $\alpha_T (n + g_T) = \alpha_T (k_T^*)^{\alpha_T - 1} = r^*$. The second equality follows from $\alpha_T k^{\alpha_T - 1} = r$ or the marginal product of capital is equal to the interest rate. This implies that $r^* < n + g_T$. Then, the transversality condition (34) is violated. Therefore, in autarky, n_N^* must be equal to n. Now suppose $n_T^* < n$. Then $1 - \lambda = L_T/L$ goes to zero and k converges to a positive constant k_N^* by (29). Then, (32) implies that k goes to zero. Contradiction. Thus, n_T^* must be equal to n.

If we plug the equation $n_N^* = n$ into (35), we can obtain the value of r^* . Collecting the results above, we obtain Proposition 1.

Proposition 1. Our autarky economy has a unique AGBGP where the following equations hold.

$$r^* = \theta + (1 - \alpha_N)(g_T - g_N) + \rho(\alpha_N g_T + (1 - \alpha_N)g_N), \tag{37}$$

$$s_N^* = 1, \tag{38}$$

$$\Pi_N^* = \Pi^* = (1 - \alpha_N) (g_T - g_N),$$
(39)

$$n_T^* = n_N^* = n, (40)$$

$$g_K^* = g_{K_T}^* = g_{K_N}^* = g_{Q_T}^* = g_{P_N Q_N}^* = n + g_T,$$

$$g_{Q_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N,$$
(41)

$$g_{O_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N,$$
 (42)

$$g_{\tilde{c}_T L}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N + \varepsilon (1 - \alpha_N) (g_T - g_N), \tag{43}$$

$$g_{\tilde{c}L}^* = g_{\tilde{c}_NL}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N, \tag{44}$$

$$\lambda^* = \frac{\frac{r^*}{\alpha_T(n+g_T)} - 1}{\frac{\alpha_N/(1 - \alpha_N)}{\alpha_T/(1 - \alpha_T)} + \frac{r^*}{\alpha_T(n+g_T)} - 1} \in (0, 1).$$
(45)

All equations follow from our key result in (40): the asymptotic growth rates of employment in the two sectors must be identical. The asymptotic growth rates for K, K_T , and K_N immediately follow from the asymptotic constancy of k, k_T , and k_N . Note that k is constant when k_T , k_N , and λ are constant by (29). The asymptotic growth rates for Q_T and Q_N can be easily obtained from production functions in (30) and (31). (43) and (44) follow from (35), (36), and (40). λ^* in (45) can be calculated in the following way. c_T goes to zero, and so does k as kconverges to a constant. Thus, by (29) and (32),

$$(n+g_T)[(1-\lambda^*)k_T^* + \lambda^* k_N^*] = (1-\lambda^*)(k_T^*)^{\alpha_T}. \tag{46}$$

Using (27), (28), and the fact that $\alpha_T(k_T^*)^{\alpha_T-1} = r^*$, the value of λ^* can be obtained.

Note that $(1 - \alpha_N)(g_T - g_N)$ is the inflation rate of nontradables, and because the share of nontradables in consumption expenditure converges to 1, it also is equal to the asymptotic CPI inflation rate. If g_T were equal to g_N , the long run equilibrium of our economy would be identical to that of the standard one-sector Ramsey economy except for the fact that the economy produces two goods. In this case, the relative price of nontradables would be constant and so would the share of nontradables in consumption expenditure. Consumption and output per capita for both goods would be growing at the same rate of g_T . r^* would be given by $\theta + \rho g_T$. Thus, any divergence of our economy from the standard Ramsey model stems from the Baumolian assumption that g_T is greater than g_N .

In our economy, s_N converges to 1, but it never becomes exactly equal to 1. Therefore, all asymptotic growth rates in Propostion 1 are never exactly realized. Note that aggregate output is equal to $Q_T + P_N Q_N$, and its asymptotic growth rate is given by $n + g_T$ by (41). Aggregate consumption expenditure $P\tilde{c}L$ also asymptotically grow at the rate of $n + g_T$ by (39) and (44). Therefore, asymptotically, aggregate output, consumption expenditure, and capital all grow at the same rate of $n + g_T$. Since $w = W/A_T$ is constant, WL also asymptotically grows at the rate of $n + g_T$. Thus, consumption-output ratio, capital-output ratio, and the labor share of income are all asymptotically constant. On the AGBGP, all Kaldor stylized facts are satisfied.

Despite Assumptions 1 and 2, which are inductive to Baumolian unbalanced growth, our autarky economy does not undergo structural change either in terms of sectoral labor allocation or value-added composition: L_N/L_T and P_N Q_N/Q_T are asymptotically constant. However, we have unbalanced growth in terms of output and consumption composition at constant prices: Q_N/Q_T and c_N/c_T converges to zero according to (41) to (44). Furthermore, $(P_Nc_N)/c_T$ grows to infinity.

Following Baumol (1967), we can call the tradable sector as the "progressive sector" and the nontradable sector as the "stagnant sector" because g_T is greater than g_N . As we have seen, the relative price of the stagnant sector good increases without bound, and its expenditure share in consumption approaches 1. Thus, the Baumol's cost disease emerges in our model. Baumol (1967) also conjectured that ulitimately, the stagnant sector would absorb all labor when its products are inelastically demanded, dragging the growth rate of the entire economy down to its slow rate of technological progress. This phenomenon, called the Baumol's growth disease, does not show up in our economy. On the AGBGP, both GDP per capita and consumption expenditure per capita in units of tradables grow at the

rate of g_T . Because CPI inflation rate is given by $(1 - \alpha_N)(g_T - g_N)$, GDP per capita and consumption expenditure per capita in units of CPI grow at the rate of $\alpha_N g_T + (1 - \alpha_N) g_N$, which is greater than g_N when $\alpha_N > 0$. The Baumol's growth disease is avoided as long as capital input is nonzero in the nontradable sector.⁵

Our results are in contrast with Acemoglu and Guerrieri (2008). They show that when $\varepsilon < 1$, the employment and value-added shares of the stagnant sector must be increasing toward unity on the AGBGP. They also show that the economy asymptotically grows at the rate of g_N , exhibiting the Baumol's growth disease. The difference from Acemoglu and Guerrieri (2008) stems from our assumption that investment uses only goods from the progressive sector. In their model, the economy produces one final good, which is used for both consumption and investment, by mixing the progressive sector good and the stagnant sector good. Because the latter two goods are poor substitutes in producing investment goods when $\varepsilon < 1$, the growth of capital stock is ultimately bound by the lower productivity growth of the stagnant sector, bringing down the growth rate of the entire economy. Our economy can avoid this phenomenon because investment does not require goods from the stagnant sector.

When we mention tradables, we have in mind agricultural and manufactured goods. Nontradables represent services like restaurants, hotels, education, healthcare, and public services. Thus, we are implicitly assuming that technology improves faster in manufacturing than in services. This assumption is not entirely satisfactory because some services, especially information technology related services, are increasingly used in investment, and also enjoy faster technological progress than manufacturing. However, we can avoid this kind of criticism by assuming that investment-related services are produced in the progressive tradable sector.

Furthermore, our specification is more desirable that that of Acemoglu and Guerrieri (2008) from the viewpoint of long-run growth theory. In their model, the price of investment goods relative to the price of consumption goods should

⁵In other words, the growth disease will occur in the case that the stagnant sector uses only labor. Baumol (1967) seemed to have this case in mind when he predicted the growth disease.

be constant because the two kinds of goods are produced by the same technology. This result conflicts with the observation that the relative price of investment goods has been secularly falling (Jones, 2016). Grossman et al. (2017) argue that the falling relative price of investment goods should be an essential feature that any satisfactory growth theory must accommodate, along with the Kaldor facts. Our closed-economy model generates the falling relative price of investment goods as well as all the Kaldor facts.

4. DYNAMICS IN A SMALL OPEN ECONOMY

We open up the economy by allowing it to trade good T and bonds with the world. The size of the economy is so small that the world interest rate r_f , which is assumed to be constant, unilaterally determines the domestic interest rate: $r = r_f$. In addition, the price of good T is always equal to 1, its world price. However, the price of nontradables P_N is internally determined.

Let us assume, until we deal with the possibility that the economy produces only nontradables, that the economy produces both tradables and nontradables. As we have seen before, (25) through (28) imply that w, p_N , k_T , and k_N are functions of r alone. Therefore, the constancy of r implies that w, p_N , k_T , and k_N are exactly constant. From (25) and (26), we immediately have $\Pi_N = (1 - \alpha_N)(g_T - g_N)$. As the price of nontradables approaches infinity, the expenditure share of nontradables in consumption increases toward 1 and CPI inflation rate converges to $(1 - \alpha_N)(g_T - g_N)$, as in autarky.

In an open economy, (32), the market-clearing condition for good T, should be modified to allow for net exports X.

$$c_T + \dot{k} + (n + g_T)k + x = (1 - \lambda)k_T^{\alpha_T},$$
 (47)

where $x \equiv X/(LA_T)$. (33), the market-clearing condition for good N remains intact.

$$c_N = \left(\frac{A_T}{A_N}\right)^{-(1-\alpha_N)} \lambda k_N^{\alpha_N}. \tag{48}$$

Now the assets held by households can include foreign bonds B in addition to domestic capital. Defining $b \equiv B/(LA_T)$, we have

$$a = k + b. (49)$$

b can be negative. The tranversality condition in (34) should be modified:

$$\lim_{t \to \infty} a \exp[-(r_f - n - g_T)t] = 0.$$
 (50)

Because (35) holds in an open economy as long as both goods are produced, we can replace r^* by r_f in the equation:

$$g_{c_N}^* = \frac{1}{\rho} \left(r_f - \theta - (1 - \alpha_N) \left(g_T - g_N \right) \right) - g_T = n_N^* - n - (1 - \alpha_N) \left(g_T - g_N \right).$$
(51)

We define r_a to be the asymptotic interest rate in autarky determined in the previous section: $r_a = \theta + (1 - \alpha_N)(g_T - g_N) + \rho[\alpha_N g_T + (1 - \alpha_N)g_N]$. (51) implies that

$$n_N^* - n = -\frac{1}{\rho} (r_a - r_f).$$
 (52)

(52) is the key equation for our analysis of an open economy.

We begin with the case $r_a > r_f > n + g_T$. The second inequality is necessary to satisfy the transversality condition (50). (52) implies that $n_N^* < n$. Thus, $\lambda^* = 0$. By (24) and (29), $n_T^* = n$ and $k^* = k_T$. The employment share of the nontradable sector converges to zero, but this does not mean that the sector will completely disappear. Nontradables should be domestically produced for domestic consumption, and the level of employment in the sector should always remain strictly positive. The asymptotic growth rates of major variables can be easily obtained from this result. The asymptotic constancy of k and k_T implies that $g_K^* = g_{K_T}^* = n + g_T$. The constancy of k_N and (52) implies that $g_{K_N}^* = n + g_T - \rho^{-1}(r_a - r_f)$. Combining these results with (30) and (31), we

obtain $g_{Q_T}^* = n + g_T$ and $g_{Q_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N - \rho^{-1} (r_a - r_f)$. Because $\Pi_N = (1 - \alpha_N) (g_T - g_N)$, $g_{P_N Q_N}^* = n + g_T - \rho^{-1} (r_a - r_f)$. Note that the growth rates of capital and output in the nontradable sector have decreased by $\rho^{-1} (r_a - r_f)$ from the autarky levels in Proposition 1. $g_{c_T}^*$ and $g_{c_N}^*$ can be obtained by plugging (52) into (35) and (36). They also decline by $\rho^{-1} (r_a - r_f)$ from the autarky levels because of a lower interest rate.

Our open economy undergoes a strong structural transformation on the AG-BGP. The expenditure share of nontradables in consumption increases toward 1, as in autarky. Simultaneously, the employment and value-added shares of the nontradable sector converge to zero. Aggregate variables also grow at different rates. $Q_T + P_N Q_N$ asymptotically grows as fast as Q_T at the rate of $n + g_T$, because the output share of nontradables converges to zero. Because $g_K^* = n + g_T$, the capital-output ratio of the economy is constant on the AGBGP. However, we can see from (51) and (52) that consumption expenditure, $\tilde{e}L = P\tilde{c}L$, asymptotically grows at the rate of $n + g_T - \rho^{-1} (r_a - r_f)$. In other words, the asymptotic growth rate of aggregate output and capital does not change when the economy is open, but the growth rate of consumption expenditure falls because of a lower interest rate. Therefore, consumption-output ratio declines toward zero on the AGBGP.

The imbalances caused by the unbalanced growth should be absorbed by the external sector of the economy. Because c_T and \dot{k} approach zero, and k converges to k_T in equation (47), $(n+g_T)k_T+x^*=k_T^{\alpha_T}$. Thus, $x^*=k_T^{\alpha_T}-(n+g_T)k_T=k_T(r_f/\alpha_T-(n+g_T))>0$, where $k_T=\alpha_T^{\frac{1}{1-\alpha_T}}r_f^{-\frac{1}{1-\alpha_T}}$ by (A3). This implies that net exports X is positive and grows at the asymptotic rate of $n+g_T$. The budget constraint (1) can be written in terms of normalized variables as:

$$\dot{a} = (r_f - n - g_T)a + w - e. \tag{53}$$

(53) and the transversality condition (50) implies that a is the present value of e-w discounted at the rate of r_f-n-g_T . Becasue \tilde{e} asymptotically grows at the rate of $g_T-\rho^{-1}(r_a-r_f)$, $e=\tilde{e}/A_T$ converges to zero. However, w is constant at $w=\phi_T^{-\frac{1}{1-\alpha_T}}r_f^{-\frac{\alpha_T}{1-\alpha_T}}$ by (A1). Therefore, a must converge to a constant equal to

$$a^* = -\frac{w}{(r_f - n - g_T)}. (54)$$

Because a = k + b and k converges to k_T , b also converges to a constant and

$$b^* = -\left(k_T + \frac{w}{(r_f - n - g_T)}\right). (55)$$

Therefore, the economy becomes a net debtor in the long run. Furthermore, the level of foreign debt grows so large that it approaches the value of entire capital stock plus the present value of all future wages. The economy will borrow against the entire value of physical capital and human wealth. Of course, this result is unlikely to hold in the real world. Domestic capital is not collateralizable in international credit markets, and future wages are difficult to borrow against even in domestic credit markets. Our result should be interpreted as identifying a strong force toward unbalanced growth in a financially integrated world in an exaggerated way. When an open economy faces a lower world interest rate, it would attempt to borrow heavily to finance increased domestic consumption and investment, and service resulting external debts by running perpetual trade surpluses. The need to run trade surpluses induces resource movements toward the tradable sector, thereby leading to structural transformation.

Appendix B shows that the AGBGP described above is locally stable and the economy produces both goods along the transition path. We summarize our results in Proposition 2.

Proposition 2. If $r_f \in (n+g_T, r_a)$ where $r_a = \theta + (1-\alpha_N)(g_T - g_N) + \rho[\alpha_N g_T + (1-\alpha_N)g_N)]$, our small open economy has a unique stable AGBGP. On the path, both goods are produced and the following equations are satisfied.

$$r = r_f, (56)$$

$$s_N^* = 1, (57)$$

$$\Pi_N^* = \Pi^* = (1 - \alpha_N) (g_T - g_N), \tag{58}$$

$$n_T^* = n, n_N^* = n - \frac{1}{\rho} (r_a - r_f),$$
 (59)

$$g_K^* = g_{K_T}^* = n + g_T, g_{K_N}^* = n + g_T - \frac{1}{\rho} (r_a - r_f),$$
 (60)

$$g_{Q_T}^* = n + g_T, g_{P_N Q_N}^* = n + g_T - \frac{1}{\rho} (r_a - r_f),$$
 (61)

$$g_{O_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N - \frac{1}{\rho} (r_a - r_f),$$
 (62)

$$g_{\tilde{c}_{T}L}^{*} = n + \alpha_{N}g_{T} + (1 - \alpha_{N})g_{N} + \varepsilon(1 - \alpha_{N})(g_{T} - g_{N}) - \frac{1}{\rho}(r_{a} - r_{f}),$$
 (63)

$$g_{\tilde{c}L}^{*} = g_{\tilde{c}_{N}L}^{*} = n + \alpha_{N}g_{T} + (1 - \alpha_{N})g_{N} - \frac{1}{\rho}(r_{a} - r_{f}),$$
 (64)

$$g_X^* = g_{(-R)}^* = n + g_T.$$
 (65)

The local transitional dynamics in terms of e, s_N , and a around the AGBGP can be calculated using Appendix B. However, we are more interested in the pattern of structural transformation during transition. To obtain the path of λ , note that $(1 - \alpha_N) p_N Q_N = WL_N$ by the property of a Cobb-Douglas production function. Dividing both sides of the equation by LA_T and using the definition of w, s_N , and e,

$$(1 - \alpha_N) s_N e = \lambda w. (66)$$

Recall that w is constant. Using (B1) and (B2) in Appendix B, we obtain

$$\frac{\dot{\lambda}}{\lambda} = \frac{\dot{s}_N}{s_N} + \frac{\dot{e}}{e} = \left(\frac{1}{\rho} - \varepsilon\right) (1 - s_N) (1 - \alpha_N) (g_T - g_N) - \frac{1}{\rho} (r_a - r_f). \tag{67}$$

If $\varepsilon \ge \rho^{-1}$, which seems empirically more relevant, λ monotonically declines to zero. If $\varepsilon < \rho^{-1}$, the path of λ can be hump-shaped: λ initially increases over time when s_N is small, and begins to fall when s_N reaches a sufficiently large value.

Using (32) and (33), and the definition of p_N , we derive

$$\frac{P_N Q_N}{Q_T} = \frac{\lambda}{1 - \lambda} p_N \frac{k_N^{\alpha_N}}{k_T^{\alpha_T}}.$$
 (68)

Note that p_N , k_N , and k_T are constant. Therefore, the value-added share of the nontradable sector moves in the same direction as the employment share. In summary, in the case where the autarky interest rate is higher than the world interest rate, both the employment and value-added shares of the nontradable sector decreases over time during transition, all the way if $\varepsilon \ge \rho^{-1}$, and at least

in later phases if $\varepsilon < \rho^{-1}$. Asymptotically, both the employment and the value-added shares decline at the rate of $\rho^{-1}(r_a - r_f)$.

We now examine the opposite case where $r_a < r_f$. (52) implies that $n_N^* > n$. However, this is impossible. Therefore, an AGBGP where the economy produces both goods does not exist in this case. On an AGBGP, the economy must stop producing tradables and import all of them from abroad in a finite time, say, from time T. In other words, for all $t \ge T$, $L_T = K_T = 0$ and $L_N = L$, $K_N = K$. In addition, the following conditions should be satisfied.

$$1 < \phi_T w^{1 - \alpha_T} \left(r_f \right)^{\alpha_T}, \tag{69}$$

$$p_N = \phi_N w^{1 - \alpha_N} \left(r_f \right)^{\alpha_N}. \tag{70}$$

$$k = k_N = \frac{w}{r_f} \frac{\alpha_N}{1 - \alpha_N},\tag{71}$$

$$c_T + \dot{k} + (n + g_T)k + x = 0,$$
 (72)

$$c_N = \left(\frac{A_T}{A_N}\right)^{-(1-\alpha_N)} \lambda k^{\alpha_N}. \tag{73}$$

(69) states that the unit cost of producing good T exceeds the price such that no firm produces good T. By (70) and (71),

$$g_{p_N} = (1 - \alpha_N) g_w, \tag{74}$$

$$g_k = g_w. (75)$$

 p_N and w are no more constant on an AGBGP despite a constant interest rate. From (16), (73), (74), and (75),

$$g_{c_N}^* = \frac{1}{\rho} (r_f - \theta) - \frac{1}{\rho} [(1 - \alpha_N) g_w^* + (1 - \alpha_N) (g_T - g_N)] - g_T$$

$$= \alpha_N g_w^* - (1 - \alpha_N) (g_T - g_N). \tag{76}$$

By (76) and the definition of r_a ,

$$g_w^* = \frac{1}{\rho \alpha_N + (1 - \alpha_N)} (r_f - r_a) \equiv \delta. \tag{77}$$

⁶The response of λ on the moment of a capital account liberalization is theoretically undetermined. Kim, Oh, and Song (2019) show that λ can jump up or down on the moment of a liberalization in a simpler model.

Unlike the previous case where the economy produces both goods, the normalized wage grows at a positive rate on the AGBGP. Thus, $\Pi_N^* = (1 - \alpha_N) g_w^* + (1 - \alpha_N) (g_T - g_N)$. We can calculate g_k^* and $g_{c_N}^*$ from (75), (76), and (77).

Because the economy completely specializes in producing nontradables, GDP is determined solely in the nontradable sector. By (73), $g_{Q_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N + \alpha_N \delta$. Adding Π_N^* to the equation, we obtain that $g_{P_N Q_N}^* = n + g_T + \delta$. GDP grows faster than in the closed economy by δ . By (75), $g_K^* = n + g_T + \delta$, and hence capital-output ratio is constant on the AGBGP. The asymptotic growth rate of consumption expenditure is equal to that of nominal nontradables consumption. Thus, $g_{\tilde{e}L}^* = n + g_T + \delta$. Thus, the consumption-output ratio is also constant on the AGBGP.

Because the economy does not produce tradables at all, it has to import them from abroad. From (15), we can show that $g_{c_T}^* = \varepsilon(1-\alpha_N)(g_T-g_N+\delta) - (1-\alpha_N)(g_T-g_N) + \alpha_N\delta$ and it is less than $g_k^* = \delta$ when $\varepsilon < 1$. Thus, c_T/k approaches zero. We also know that k/k approaches δ . Then, we can derive from (72) that

$$\frac{x^*}{k^*} = -(n + g_T + \delta). \tag{78}$$

Therefore, on the AGBGP, total imports (-X) grow at the rate of $g_K^* = n + g_T + \delta$. To satisfy the transversality condition, we impose the condition $r_f > n + g_T + \delta$ or

$$(1 - \rho) \alpha_N [r_f - (n + g_T)] < r_a - (n + g_T). \tag{79}$$

(79) is always satisfied for any $r_f > r_a$ when $\rho \ge 1$. Using (53), (66), and (71), we can show that

$$\frac{a^*}{k^*} = \frac{r_f}{r_f - n - g_T - \delta}. (80)$$

Because a = b + k,

$$\frac{b^*}{k^*} = \frac{n + g_T + \delta}{r_f - n - g_T - \delta}.$$
(81)

The economy becomes a net creditor on the AGBGP, and both physical capital and foreign assets grow at the rate of $n + g_T + \delta$. (81) implies together with (78) that the amount of foreign assets is equal to the present value of future trade deficits. Faced with a higher world interest rate, the economy cuts down on consumption and investment, and accumulates foreign assets. The amount of foreign assets becomes so large that it can finance all future imports of tradables, which the economy does not produce at all. Again, our model captures the influence of international capital mobility on structural transformation in quite a dramatic manner.

We summarize our results in Proposition 3.

Proposition 3. If $r^f > r_a$ with $(1 - \rho) \alpha_N \left[r^f - (n + g_T) \right] < r_a - (n + g_T)$, our small open economy produces only nontradables on the AGBGP, satisfying the following equations.

$$r = r_f, (82)$$

$$s_N^* = 1, \tag{83}$$

$$\Pi_N^* = \Pi^* = (1 - \alpha_N)(g_T - g_N + \delta),$$
 (84)

$$L_T^* = 0, K_T^* = 0, L_N^* = L, K_N^* = K.$$
 (85)

$$g_K^* = g_{K_N}^* = g_{P_N Q_N}^* = n + g_T + \delta,$$
 (86)

$$g_{Q_N}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N + \alpha_N \delta, \tag{87}$$

$$g_{\tilde{c}_{T}L}^{*} = n + \alpha_{N}g_{T} + (1 - \alpha_{N})g_{N} + \varepsilon (1 - \alpha_{N})(g_{T} - g_{N} + \delta) + \alpha_{N}\delta, \quad (88)$$

$$g_{\tilde{c}L}^* = g_{\tilde{c}_NL}^* = n + \alpha_N g_T + (1 - \alpha_N) g_N + \alpha_N \delta, \tag{89}$$

$$g_{(-X)}^* = g_B^* = n + g_T + \delta,$$
 (90)

where
$$\delta = \frac{1}{\rho \alpha_N + (1 - \alpha_N)} (r_f - r_a)$$
.

Comparing with Proposition 2, we note that on the AGBGP, output, capital, and consumption all grow faster when the economy is completely "deindustrialized" with a higher world interest rate than when the economy has a dominant manufacturing sector with a lower world interest rate. This result may look counterintuitive, but it follows from the Ramsey rule of consumption: consumption grows faster with a higher interest rate.

Before the economy reaches a point where it begins to specialize at time T, it produces both goods. As long as the economy produces both goods, transitional

dynamics is still governed by the same system as before. Thus, (67) and (68) are still satisfied. λ follows (67) before it reaches unity at time T. When $\varepsilon \leq \rho^{-1}$, λ monotonically increases before it reaches unity because $r_f > r_a$. When $\varepsilon > \rho^{-1}$, λ may initially decline when s_N is small, but increases toward unity after s_N reaches a sufficiently large value. Again, (68) implies that the value-added share of the nontradable sector moves in the same direction as the employment share.

5. CONCLUSION

This study emphasizes the role of international capital mobility in determining the pattern of structural transformation. It is now well understood that goods trade can significantly affect the pattern of structural transformation through the channel of comparative advantages. This paper adds another international dimension to the literature on structural transformation by showing that in a financially integrated world, differential rates of technological progress across countries can exercise strong influence on structural transformation through intertemporal substitution of consumption. We also find that the force toward unbalanced growth is so strong that in the long run, it dominates the trend toward deindustrialization induced by the Baumol's cost disease.

Although our model serves to highlight the role of international capital mobility and the nontradable sector in structural changes, its predictions, especially about foreign asset positions, need to be modified to be matched up with empirical observations on international capital flows. Introducing finite horizon, imperfect capital mobility, or convergence in technology is a promising route to making our model more compatible with data. Incorporating "wedges" in saving and investment could be another. These exercises should be our agenda for future research.

. APPENDIX A

If the economy produces both tradables and nontradables, w, p_N , k_T , and k_N can be expressed as functions of r alone:

$$w = \phi_T^{-\frac{1}{1-\alpha_T}} r^{-\frac{\alpha_T}{1-\alpha_T}},\tag{A1}$$

$$p_N = \phi_T^{-\frac{1-\alpha_N}{1-\alpha_T}} \phi_N r^{-\frac{\alpha_T - \alpha_N}{1-\alpha_T}}$$
 (A2)

$$k_T = \alpha_T^{\frac{1}{1-\alpha_T}} r^{-\frac{1}{1-\alpha_T}},\tag{A3}$$

$$k_N = \frac{\alpha_N/(1-\alpha_N)}{\alpha_T/(1-\alpha_T)} \alpha_T^{\frac{1}{1-\alpha_T}} r^{-\frac{1}{1-\alpha_T}}, \tag{A4}$$

where $\phi_i = \alpha_i^{-\alpha_i} (1 - \alpha_i)^{-(1 - \alpha_i)}$

. APPENDIX B

Transitional dynamics toward the AGBGP after the economy opens up from an autarkic position can be obtained from the dynamic system composed of (B1), (B2), and (B3).

$$\dot{e} = \left[-\frac{1}{\rho} (r_a - r_f) - \left(1 - \frac{1}{\rho} \right) (1 - s_N) (1 - \alpha_N) (g_T - g_N) \right] e, \tag{B1}$$

$$\dot{s}_N = (1 - \varepsilon) \, s_N \, (1 - s_N) \, (1 - \alpha_N) \, (g_T - g_N) \,,$$
 (B2)

$$\dot{a} = (r_f - n - g_T)a + w - e. \tag{B3}$$

(B2) is autonomous in s_N . Because s_N is a function of $P_N = (A_T/A_N)^{1-\alpha_N} p_N$ and p_N is constant, $s_N(0)$ is a predetermined variable. Starting from this initial value, s_N increases toward 1 along the path defined by (B2), independently of e and e is a jumping variable whose initial value is not predetermined, and e is a state variable with e0 is the stock of capital at the moment of capital market opening.

 $^{^{7}}k$ can jump up at time 0 with capital inflows, but then b should jump down by the same amount to finance investment. k+b does not change from k(0) at the moment of capital market opening.

For the AGBGP in Proposition 2 to be stable, e, s_N , and a should converge to $e^* = 0$, $s_N^* = 1$, and $a^* = -\frac{w}{r_f - n - g_T}$, respectively. Linearizing the system around these asymptotic values, we obtain:

$$\begin{bmatrix} \dot{e} \\ \dot{s}_N \\ \dot{a} \end{bmatrix} = \begin{bmatrix} -\rho^{-1} (r_a - r_f) & 0 & 0 \\ 0 & -(1 - \varepsilon) \Pi_N & 0 \\ -1 & 0 & r_f - n - g_T \end{bmatrix} \begin{bmatrix} e - 0 \\ s_N - 1 \\ a - a^* \end{bmatrix}, \quad (B4)$$

where $\Pi_N = (1 - \alpha_N)(g_T - g_N)$. The three eigenvalues of the system can be easily obtained: $-\rho^{-1}(r_a - r_f)$, $-(1 - \varepsilon)\Pi_N$, and $r_f - n - g_T$. Two eigenvalues are negative and one eigenvalue is positive. Because we have two state variables and one jumping variable, the system is locally saddle-point stable.

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